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Daley, H; Iachello, F

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## ALPHA-CLUSTERING IN HEAVY NUCLEI

H. DALEY

*Physics Department, University of Arizona, Tucson, AZ 85721, USA*

and

F. IACHELLO

*A.W. Wright Nuclear Structure Laboratory<sup>1</sup>, Yale University, New Haven, CT 06511, USA  
and Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, The Netherlands*

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We present calculations of the spectra of the  $^{88}\text{Ra}$  and  $^{90}\text{Th}$  isotopes within the framework of an algebraic approach to clustering in nuclei.

It has been suggested recently [1,2], on the basis of certain anomalies observed in the structure of the actinide nuclei, that  $\alpha$ -clustering may play a role in heavy nuclei as important as that played in light nuclei. In studying  $\alpha$ -clustering in heavy nuclei a major difficulty is that one of the fragments is usually not spherical and thus one must treat explicitly the interaction of the clustering degrees of freedom with the quadrupole degrees of freedom that give rise to the deformation. In ref. [1] an approach for treating this problem was suggested and in this letter we report the first results obtained using it. Although the approach discussed in ref. [1] is still semiphenomenological and thus contains several parameters, the conclusion we draw from the comparison between our calculations and the experimental data is that all quantities measured at present in the light actinides (neutron number  $< 142$ ) are consistent with an  $\alpha$ -clustering interpretation.

In order to describe  $\alpha$ -clustering states in heavy nuclei, two algebraic structures were introduced in ref. [1], one described by the group  $U(6)$  and generated by  $(s, d)$  bosons, and one described by the group  $U(4)$  and generated by

$(s^*, p^*)$  bosons. The former accounts for the quadrupole degrees of freedom responsible for the deformation [3], while the latter accounts for the dipole degrees of freedom responsible for clustering [4]. The corresponding hamiltonian is written as

$$H = H_d + H_p + V_{dp}, \quad (1)$$

where  $H_d$  describes the deformation,  $H_p$  the clustering and  $V_{dp}$  their interaction. In the spirit of keeping the calculations as simple as possible we have used the following parametrizations,

$$\begin{aligned} H_d &= \epsilon_d n_d + \kappa_d Q_d \cdot Q_d + (\kappa' + \frac{3}{8}\kappa_d) L_d \cdot L_d, \\ H_p &= \epsilon_p n_p + \alpha_p n_p(n_p - 1) + \kappa' L_p \cdot L_p, \\ V_{dp} &= \kappa Q_d \cdot Q_p + 2\kappa' L_d \cdot L_p, \end{aligned} \quad (2)$$

where  $n_d$ ,  $n_p$  are the number operators for  $d$  and  $p^*$  bosons,  $Q_d$ ,  $Q_p$  their quadrupole operators and  $L_d$ ,  $L_p$  their angular momentum operators. In this parametrization, the deformation is described by a situation intermediate between that of an anharmonic vibrator  $\{U(5)$  limit of ref. [3] $\}$  and that of an axially symmetric rotor  $\{SU(3)$  limit of ref. [3] $\}$ , while the clustering is described by oscillations of the  $\alpha$ -particle relative to the remaining part of the nucleus  $\{U(3)$  limit of ref. [4] $\}$ . We also tried parametrizations

<sup>1</sup> Permanent address.

in which the clustering was described by a rigid molecular-like structure {O(4) limit of ref. [4]}, but those parametrizations did not appear to describe the data. The values of  $\kappa_d$ ,  $\kappa'$ ,  $\alpha_p$  and  $\kappa$  were kept constant for a set of isotopes, while the values of  $\epsilon_d$  and  $\epsilon_p$  were varied. This variation is shown in table 1 and discussed further below. The constant values were chosen to be

$$\kappa_d = -0.035 \text{ MeV}, \quad \kappa' = 0.010 \text{ MeV},$$

$$\alpha_p = 1.0 \text{ MeV}, \quad \kappa = -0.090 \text{ MeV},$$

for  $^{88}\text{Ra}$  and

$$\kappa_d = -0.020 \text{ MeV}, \quad \kappa' = 0.009 \text{ MeV},$$

$$\alpha_p = 1.0 \text{ MeV}, \quad \kappa = -0.090 \text{ MeV},$$

for  $^{90}\text{Th}$ .

An additional complication, already known from light nuclei, is that different configurations co-exist in the same nucleus. An example is shown in fig. 9 of ref. [2]. In the calculations reported here, only two configurations were kept, the configuration with *no*  $\alpha$ -clusters (denoted by  $0\alpha$ ), and the configurations with *one*  $\alpha$ -cluster (denoted by  $1\alpha$ ). Other, more complex, configurations ( $2\alpha$ ,  $3\alpha$ , ...) could be added, if needed. The parameters describing the various  $\alpha$ -clustering configurations could, in general, be different. We have kept them equal, with the exception of the parameter  $\epsilon_d$  which we have taken to be  $\epsilon_d = 0$  for the  $1\alpha$  configuration, while varying it as shown in table 1 for the  $0\alpha$

configuration. This choice reflects the fact that we expect the  $1\alpha$  configuration to be more deformed than the  $0\alpha$  configuration, and thus closer to the SU(3) limit of ref. [3]. This limit is reached precisely when  $\epsilon_d = 0$ . Our choice is also consistent with experiment since the rotational bands that we associate with the  $1\alpha$  configuration appear to have a moment of inertia somewhat larger than that of the bands of the  $0\alpha$  configuration.

In order to perform the configuration mixing calculation we must specify, in addition to the hamiltonian of eq. (2), the boson numbers for the two configurations ( $0\alpha$  and  $1\alpha$ ), the energy difference  $\Delta\alpha = E_{1\alpha} - E_{0\alpha}$ , and the strength of the mixing interaction that we take, again for

Table 1

Values of the parameters  $\epsilon_d$ ,  $\epsilon_p$  and  $\Delta\alpha$  (all in MeV) used in the calculation

Neutron number	$^{88}\text{Ra}$			$^{90}\text{Th}$		
	$\epsilon_d$	$\epsilon_p$	$\Delta\alpha$	$\epsilon_d$	$\epsilon_p$	$\Delta\alpha$
128	0.90	0.00	1.20	0.90	0.20	1.20
130	0.70	0.00	1.10	0.70	0.20	1.10
132	0.50	0.00	0.85	0.50	0.20	0.85
134	0.30	0.00	0.30	0.30	0.16	0.68
136	0.10	0.00	-0.57	0.10	0.16	0.00
138	0.00	0.06	-1.25	0.00	0.25	-0.55
140	0.00	0.44	-1.60	0.00	0.49	-0.70
142	0.00	0.87	-1.80			

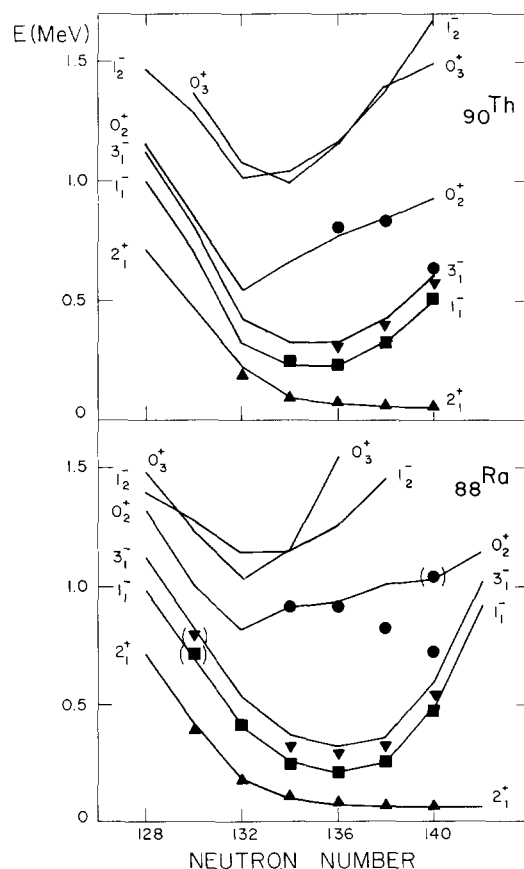


Fig. 1. Comparison between calculated and experimental energy levels in  $^{88}\text{Ra}$  and  $^{90}\text{Th}$ .

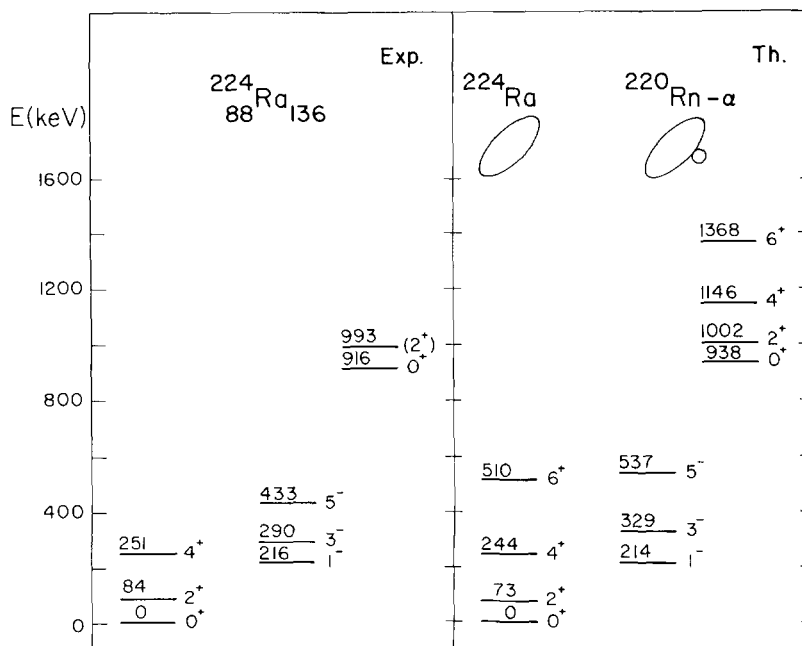


Fig. 2. Configuration mixing in  $^{224}\text{Ra}$ . The  $1\alpha$  configuration is denoted by  $^{220}\text{Rn}-\alpha$ .

simplicity, of the form

$$V_{\text{mix}} = \gamma(s^{\dagger 2}s^{*2} + s^{* \dagger 2}s^2). \quad (3)$$

We count the number of bosons as follows. For the configuration  $0\alpha$  we take the number of pairs outside the major closed shells (82 and 126 in this case), as usual [3]. We call this number  $N$ . For the configuration  $1\alpha$  we take the number of  $(s, d)$  bosons to be  $N - 2$  and the number of  $(s^*, p^*)$  bosons to be 2. This means that we consider only up to two quanta of oscillation of the cluster [4]. There is, in principle, no reason why one should limit oneself to two quanta. We have done this for simplicity, even though the computer program that we have written can deal with larger values. Finally, we kept  $\gamma$  constant in the calculations ( $\gamma = -0.080$  MeV) and varied  $\Delta_\alpha$  as shown in table 1.

The results of our calculations of the energy spectra are shown in fig. 1. In fig. 2 we take one case,  $^{224}\text{Ra}$ , and separate it into its various components. In the  $\alpha$ -clustering interpretation, the ground state  $K^\pi = 0^+$  band is described by the  $0\alpha$  configuration with, on the average, about 10% admixture of the  $1\alpha$  configuration. This

admixture is responsible for the ground state to ground state  $\alpha$ -decay. The lowest  $K^\pi = 0^-$  band is described by the one-phonon dipole oscillation of the cluster. The energy of this oscillation, which normally should be higher than that of the band head of the  $1\alpha$  configuration, is lowered by the coupling to the deformation. The  $K^\pi = 0^-$  band describes an oscillation along the symmetry axis. Conversely, the  $K^\pi = 1^-$  band describes an oscillation perpendicular to the symmetry axis. Its energy is raised by the coupling to the deformation and we calculate it at  $\approx 1300$  keV (see fig. 1). The lowest excited  $K^\pi = 0^+$  band at 916 keV is described by the  $1\alpha$  configuration with about 10% admixture of the  $0\alpha$  configuration. This admixture accounts for the observed  $\alpha$ -decay hindrance factor ( $\approx 6$ ). The interpretation of the spectrum of  $^{224}\text{Ra}$  shown in fig. 2 is similar to that of the spectrum of  $^{20}\text{Ne}$  shown in fig. 9 of ref. [2]. It is different from the interpretation in terms of octupole vibrations [5] in the nature of both the lowest excited  $K^\pi = 0^-$  and  $K^\pi = 0^+$  bands. We have calculated in addition to energies,  $\alpha$ -decay probabilities (mentioned above),  $B(E1)$  and

$B(E2)$  values, all of which appear to be consistent with the data. It would be interesting to perform similar calculations (in particular those of the  $\alpha$ -decay widths) within the framework of the octupole interpretation. A comparison with the experimental data may then allow one to decide which of the two interpretations is more appropriate.

In conclusion, the fact that our calculations are all consistent with the data, suggests that  $\alpha$ -clustering may indeed play an important role in the structure of the light actinides. This suggestion is strengthened by the recently measured large  $E1$  matrix elements between negative and positive parity states in  $^{218}\text{Ra}$  [6] and  $^{222}\text{Th}$  [7]. The scope and limitations of the  $\alpha$ -clustering role remains to be determined by further experiments. An important result of our calculation is that the  $\alpha$ -clustering states appropriate to this region appear to be those corresponding to dipole vibrations of the cluster {U(3) limit of ref. [4]} rather than those corresponding to rigid molecular-like dipole deformations {O(4) limit of ref. [4]}. We also note that the role played by  $\alpha$ -clustering in the heavy actinides (neutron number  $> 142$ ) may be different from that in the light actinides as was discussed here. For example, an extrapolation of our calculations to this region shows that the negative parity states of the  $\alpha$ -clustering configuration move to higher and higher excitation energy with increasing neutron number. For certain values of the neutron number they may thus attain energies comparable to those of the octupole vibrational states. The interference and coupling with these states could no longer be neglected if that is the case. Finally, other

clustering configurations might become important in the heavier isotopes, as evidenced by the occurrence of *two* excited  $0^+$  bands in  $^{226}\text{Ra}$  and  $^{228}\text{Ra}$  strongly populated in (d,  $^6\text{Li}$ ) reactions [8].

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